A Graph Model of Alignment in Multilog

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***Abstract*—Justified by empirical-driven reasoning, this papers extends a graph-theoretical model of lexical alignment in dialog towards an initial conceptual model for alignment in multilog (three-and-more speaker discourse). Underway, the monotonic increasing alignment function is counterbalanced by an unlearning function which allows for capturing the drifting apart of language use. Thus, the paper contributes to alignment in communication in two respects: (i) by acknowledging "anti-alignment" alongside with the common notions of similarity or repetition, and (ii) by providing a theoretical starting point for quantifying alignment in multilog.**

I. Introduction

Alignment of interlocutors' representations on all linguistic levels is a well-observed phenomenon in natural language dialog. On the phonetic level, for instance, one interlocutor may accommodate his pronunciation of a certain word to that of the other interlocutor [1]; on the semantic level, interlocutors converge onto a common reference frame in order to locate objects [2]. In [3] a network model called TiTAN has been developed that captures lexical alignment in dialog, that is, alignment of word usages [4]. Human conversation, however, consists not only of dyadic interactions. Often, communication involves more than two people. In order to account for alignment in three- or more-party discourse, in Section II we not only scale-up the TiTAN model from dialog to multilog, but also abstract away from a particular linguistic level of alignment. A straightforward extension from dialog to multilog is well-founded but only if multilog is basically of the same communication type as dialog, that means in particular, of a communication type that also obeys the alignment theory originally developed for dyads [5]. The greater number of participants and communicative roles in multilog as compared to dialog may well lead to a conversation type on its own. The sparse empirical literature on the nature of multilogs suggests that the differences between two-party and multi-party discourse are indeed superficial. Dialog and multilog seem to be variants of one kind of conversation, the latter just being more complex due to the greater number of participants. The main arguments in support of the equality of dialog and multilog are the following (see also [6]):

- "two-party interactions are dominant in three-party dialogs" [7, p. 587]. That means that the formation of dyads, which adhere to the alignment theory, is a generic feature of "trilogs".
- In both multilog and dialog is the type of conversation determined via conversational goals and interlocutors' intentions [8, p. 213]. This means that the structure of multilog, like that of dialog, can in principle be captured by intention-based models like that of [9] (we exploit this fact in Section II).

Justified by these empirical hints, we develop a model of alignment in multilog that scales up from alignment in dialog. Section II generalizes two-layer to multi-layer networks. In particular, the multi-layer architecture complicates the creation and weighting of intra- as well as interpersonal alignment relations between interlocutors. These topics are dealt with in subsections II-B and II-C, respectively. There, the function that regiments linkage between multilog participants is parameterized in such a way that it not only accounts for alignment of language use, but also for its drifting apart. Building upon the multi-layer generalization, Section III presents a first conceptual measure of alignment in multilog. The paper concludes with a brief summary and outlook in Section IV.

II. Generalizing Network Models of Alignment

In [3], [10], [11], we introduced and evaluated time series of two-layer networks – the so called TiTAN-series – as models of dyadic conversations that can be decomposed into bipartite graphs together with part-internal networks.[1] Generally speaking, simple graphs can always be represented in this way. Thus, what makes two layer networks special is not their graph-theoretical status. Rather, it is their law-like generation as a result of, for example, dyadic conversations. This is explained in detail by [3] who introduce a series of models of random graphs that provide baselines for measuring this law-like behavior.

Starting from this network model, we generalize the TiTAN-model in the following respects:

1) Unlike our work on *two-layer* TiTAN-series, we additionally focus on *multilogs* [13], that is, conversations

[1] An example of a two-layer network is WordNet [12] whose parts are spanned by words and synsets, respectively, which have graph structure on their own.

Table I
TITAN SERIES AS A MODEL OF NATURAL LANGUAGE CONVERSATIONS.

	network	model
dialog	two-layer network	[3]
multilog	multi-layer network	Section II-A

of more than two interlocutors. According to this abstraction, we conceive TiTAN-series that can be decomposed into the dialog lexica of several conversational partners.
2) Other than in *lexical* TiTAN-series, we abstract from the linguistic resolution of alignment. According to this abstraction, we take the level of alignment as a parameter of our model.
3) Other than in TiTAN-series based on priming in terms of the repetition of linguistic units, we abstract from the source of priming. According to this abstraction, we introduce a parameter in terms of a measure that can be instantiated by a model of repetition, functional, semantic or structural similarity or even of relatedness (in the sense of [14]).

Table I outlines how this approach is related to its predecessor. By realizing these generalizations, we extend the TiTAN-model as we account for time series of conversational lexica of linguistic units (whether on the lexical level or on other levels) in which for every time point a so called *multi-layer network* is emitted that is distributed among k conversational partners. More specifically, we start from an abstract model of a multilog to which $k \geq 2$ agents participate. Regarding this scenario we ask for intra- and interpersonal alignment where the latter goes beyond dyads. Following the reasoning outlined in the introduction we see alignment is a variable whose dynamics is enslaved by the interplay of all conversational partners such that their overall alignment degree varies at different points in time and is distributed among subgroups of partners. Thus, we assume that turns have the potential to reduce the amount of alignment already reached in the group. We also assume that in multi-part conversations, alignment can be distributed among the partners non-uniformly (see Figure 4 below for an extract of the spectrum of possibilities in four-part conversations). >From this perspective, the formation of alignment can be envisioned as a process where each linguistic output of each agent may reduce, confirm or enforce his alignment with his conversational partners (other-alignment) as well as with himself (self-alignment). In the former case, we speak of *interpersonal* alignment, in the latter of *intra-personal* alignment. Further, we conceive the reduction of the strength of a link as a process of *unlearning*, while in the opposite case we speak of *learning* the link. In this sense, alignment is a dynamic process to which several processes of learning and unlearning contribute that are distributed among the conversational partners. Ideally, in the sense of the *Interactive Alignment Model* [5], all partners are aligned at the end of their conversation intra- and interpersonally to a certain degree. So far, there is not much known about measuring alignment. This holds all the more for multilogs. Thus, the present paper can be read to prepare the way to such measurements.

Note that we did not specify the linguistic level of alignment in this model, whether on the lexical, syntactic or any other level. The reason is that we keep this level as a parameter of our model.[2] In order to distinguish this level from the context of counting, we use the following terminology: the level of alignment under consideration will be referred to as the *level of counting units*, while the frame in which occurrences of these units are counted is called the *frame of counting*. We may refer, for example to the lexical level as the level of counting and to turns as the frame of counting. In this case, co-occurrences of lexical tokens are counted in turns and are compared for their relatedness or similarity to get information about the alignment of conversational partners. Alternatively, we may count phrase structures on the phrasal level using adjacency pairs as frames of counting. Note that in data-oriented parsing, it is not uncommon to count phrase structures as tokens of the same phrase structure type [15]. In any event, we leave it open how the level and frame of counting are instantiated.

In the following subsections, we formalize this blueprint step by step.

A. Multi-layer Networks

Generally speaking, a *k-layer network* is a graph $G = (V, E)$ that can be decomposed into a k-partite subgraph $G_0 = (V_0, E_0) \subseteq G$ together with a set $\mathbb{G} = \{G_1, \ldots, G_k\}$ of graphs all of which fulfill the following constraints:[3]

- $G_0 = (V_0, E_0)$ is a k-partite subgraph of G whose vertex set $V_0 = V$ is partitioned into k non-empty disjunct subsets $V_1, \ldots, V_k$ and whose edge set $E_0 \subseteq E$ is partitioned into $k(k-1)/2$ disjunct but not necessarily empty subsets $E_{\{1,2\}}, \ldots, E_{\{k-1,k\}}$ such that every edge $\{v, w\} \in E_{\{i,j\}}, \{i, j\} \in [\{1, \ldots, k\}]^2$, ends at vertices $v \in V_i$ and $w \in V_j$, respectively.
- Every graph $G_i = (V_i, E_i) \in \mathbb{G}$ is a subgraph $G_i \subseteq G$ of G such that V_i is the vertex set of the ith part of G_0 and $E_i \subseteq [V_i]^2$.

Thus, multi-layer networks are partitioned into parts that are subgraphs in themselves and whose vertex sets span a k-partite graph. In order to capture this dual nature of k-layer networks we denote them by means of $k + 1$-tuples:

$$\hat{G} = \langle G_0, G_1, \ldots, G_k \rangle \tag{1}$$

[2]Note also that we do not deal with multilevel alignment and, thus, not with percolation between different levels. This will be the object of further generalizations of TiTAN-series in the future.

[3]See [16] for a similar notion of multi-layer networks.

We call any subgraph G_i of $\hat{G}$ the ith layer of $\hat{G}$ and use, by analogy to [3], the projection function $\pi_{\hat{G}}\colon \{0,1,\ldots,k\} \to \mathbb{G} \cup \{G_0\}$ to project $\hat{G}$ onto the ith layer or the k-partite graph G_0, respectively:

$$\pi_{\hat{G}}(i) = G_i \tag{2}$$

The vertex and edge set of any such projection is denoted by $V(\pi_{\hat{G}}(i)) = V_i$ and $E(\pi_{\hat{G}}(i)) = E_i$, $i \in \{0,1,\ldots,k\}$. In terms of our graph model of multilogs, $\pi_{\hat{G}}(i)$ is a graph model of items of whatever linguistic, communicative provenance of the ith interlocutor, while G_0 represents the bridging set of items that the interlocutors use to establish a common communication space in the case that $\hat{G}$ is labeled (see below). Finally, $\hat{G}$ is the overall multilog lexicon of the conversation of all interlocutors. Thus, edges in G_0 denote *interpersonal* associations among linguistic items, while edges in any layer graph G_i, $i \in \{1,\ldots,k\}$, denote *intrapersonal* associations. Note that we use the notion of a set of linguistic items in the sense of an inventory of the types used during the conversation under consideration.

So far, we did not consider edge weighting and vertex labeling. These are necessary to map the strength of linguistic associations and the sharing of commonly used linguistic items. Moreover, TiTAN-series evolve as a function of time. Having this in mind, we define *time-aligned multi-layer networks* $\hat{G}_t$ as follows:

$$\hat{G}_t = \langle G_{0_t}, G_{1_t}, \ldots, G_{k_t} \rangle \tag{3}$$

such that any subgraph G_{i_t}, $i \in \{0,1,\ldots,k\}$, is defined as follows:

$$G_{i_t} = (V_{i_t}, E_{i_t}, \mu_{i_t}, \mathcal{L}_i) \tag{4}$$

where $t \in \mathbb{N}$ represents the point in time $t \in \{1,\ldots,n\}$ at which G_{i_t} is (re)built as a layer of $\hat{G}_t$. We assume that $\mathcal{L}_i\colon V_i \to L_i$ is a bijective function that labels all vertices V_i with the items they represent such that there are no equally labeled vertices within the same layer. However, $\mathcal{L} = \cup_{i=1}^{k}\mathcal{L}_i$ is a surjective function $\mathcal{L}: V_0 \to \cup_{i=1}^{k} L_i = L = \{\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \ldots\}$ such that vertices $v \in V_i, w \in V_j$, $i \neq j$, may have the same label, that is, $\mathcal{L}(v) = \mathcal{L}_i(v) = \mathcal{L}_j(w)$.

In what follows, we introduce the function

$$\mu_{i_t}\colon E_{i_t} \to \mathbb{R}_0^+ \; ; \; i \in \{0,\ldots,k\} \tag{5}$$

as the central parameter of our model. More specifically, μ_{i_t} is used to model the learning and unlearning of intra- and interpersonal links, respectively. It is used to recalculate (i.e., to decrement, increment or to keep constant) the associative strength of intra- and interpersonal links as a function of time. Since μ_{i_t} depends on time, it allows us to define a time series

$$\hat{\mathbb{G}} = \{\hat{G}_0, \ldots, \hat{G}_t, \ldots \hat{G}_l\} \tag{6}$$

of time-aligned multilayer networks $\hat{G}_t$, $t \in \{0,\ldots,l\}$, that are emitted by incrementing t. Like [3], we assume that this happens whenever a linguistic output of the focal level of alignment is produced. An alternative is to equalize time points with turns.[4]

In order to introduce the generation of this time series, we start from the dialog model of [9]. That is, we assume that any multilog $\mathcal{M}$ has an overall *discourse purpose* (DP) $P(\mathcal{M})$ such that $\mathcal{M}$ is partitioned into consecutive discourse segments $S_1,\ldots,S_n$ each of which has a *discourse segment purpose* (DSP) $P(S_i)$, which specifies the contribution of S_i to $P(\mathcal{M})$. Any such segmentation may produce graph-like structures. For simplicity, we concentrate on such segments in tree-like segmentations that are located on the same level by having the same distance to the root. For example, one can segment using turns. Another segmentation may consider adjacency pairs as segments. The segments of any such segmentation are referred to as the frame of counting while units on a certain level of linguistic constituents of these frames are referred to as counting units (see above). Throughout this paper, the segmentation is seen to be ordered by a linear order relation $\prec \subset \{S_1,\ldots,S_n\}^2$, where $S_i \prec S_j \Leftrightarrow i < j$ so that we can write $(S_1,\ldots,S_n)$. We assume that $(S_1,\ldots,S_n)$ corresponds to a purpose or topic-related organization of $\mathcal{M}$ such that each segment S_i is uniquely mapped onto its discourse purpose or topic. This mapping is modeled by means of the surjective function $\theta : \{S_1,\ldots,S_n\} \to \mathbb{T}$, where $\mathbb{T} = \{T_1,\ldots,T_m\}$, $m \ll n$, is a set of purpose or topic labels. If $\mathbb{T}$ denotes a set of topics, semantic alignment is addressed; otherwise, functional alignment is in the focus.

Now, look at any segment S_i, $i \in \{1,\ldots,n\}$, as generated by one or more conversational partners each of which generates a (not necessarily continuous) subsequence of constituents of S_i of the focal level of counting (e.g., lexical, phrasal or syntactic units). In order to relate any of these constituents a_t to $\theta(S_i)$, we write $\theta(a_t) = \theta(S_{a_t})$, where a_t is a token of some type (e.g., lemma) $\tau(a_t) = \mathfrak{a} \in L$ that labels some vertex $v \in V_i$ in the lexicon of agent $A_i \in \mathcal{A}$ who generated a_t at time t as part of the segment $S(a_t) = S_{a_t} = S_i$ where $S\colon \{a_1,\ldots,a_l\} \to \{S_1,\ldots,S_n\}$ maps tokens to the segments to which they belong. Note that we assume a function $\tau : \mathcal{A}(\mathcal{M}) \to L$ from tokens to their types where $\mathcal{A}(\mathcal{M}) = \{a_1,\ldots,a_l\}$ is the set of all tokens of $\mathcal{M}$ in terms of the focal level of linguistic resolution. Further, by $A_i(\mathcal{M}) \subset \mathcal{A}(\mathcal{M})$ we denote the subset of tokens that are produced by agent $A_i \in \mathcal{A}$ where $\mathcal{A} = \{A_1,\ldots,A_h\}$ is the set of all conversational partners that participate in $\mathcal{M}$. In this line of thinking, we can represent $\mathcal{M}$ as a time series $(a_1,\ldots,a_l)$ of tokens each of which is produced by a single agent as part of the corresponding segment S_{a_t} that has a single topic or purpose $\theta(S_{a_t})$. Note that in the case of simultaneous

[4]Note that while we assume that associations change over time, sign vehicles are seen to be stable. Note also that this is not generally true for all multilogs and can be seen as a starting point of a further generalization of the model.

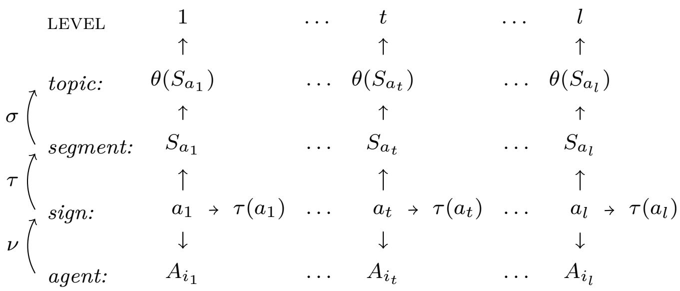

Figure 1. Multi-layer multilog network.

language productions by different partners we assume a lexicographical order that guarantees this linearity.[5] In a nutshell, we distinguish several functionally related levels as depicted in Figure 1.

Starting from these preliminaries, a multi-layer network $\hat{G}_t$ can be spanned at time t by accounting for interpersonal and intrapersonal links as follows.

B. Interpersonal Links

At time t, let agent $A_i \in \mathcal{A}$, $i \in \{1, \ldots, k\}$, be in the role of the speaker, that is, of the initiating conversational partner (ICP) (cf. [9]). Further, let a_t be the linguistic unit produced by A_i at t as part of the focal level of counting, where a_t is a token of the linguistic type $\mathfrak{a}$ of the focal level of linguistic resolution (whether complex or not) in the focal segment S_{a_t} and where $\mathfrak{a}$ labels vertex $v \in V_i$. In other words, $A_i \in \mathcal{A}$ uses $\mathcal{L}(v) = \mathcal{L}_i(v) = \mathfrak{a} \in L$ to express the purpose or topic $\theta(S_{a_t})$ of segment S_{a_t} by manifesting the type $\mathfrak{a} \in L$ by means of the token $a_t \in \mathcal{A}(\mathcal{M})$. Now, for any agent $A_j \in \mathcal{A}$, $A_j \neq A_i$, we proceed as follows: if A_j instantiated the same type $L \ni \mathfrak{b} = \mathfrak{a}$ or some type $\mathfrak{b}$ that is related or similar to $\mathfrak{a}$, that is, $\mathfrak{a} \sim \mathfrak{b}$, and if $\mathfrak{b}$ is the label of vertex $w \in V_j$ that has been instantiated by means of the token a_p, $p < t$, $\tau(a_p) = \mathfrak{b}$, as part of the segment

$$S_{a_p} \preceq S_{a_t} \tag{7}$$

to speak about the same purpose or topic $\theta(Sa_t)$, we face the kind of basic information that we explore to generate a so called *interpersonal link* between A_i and A_j. This is done using the initial weight $\sigma(\mathfrak{a}, \mathfrak{b})$ based on some similarity function [17] σ (see below). Otherwise, if such an edge between A_i and A_j already exists in $G_{0_{t-1}}$, its weight is incremented by means of the learning function $\mu_t \colon \mathbb{R}_0^+ \to \mathbb{R}_0^+$, which is the central parameter of our model (see below). In this paper, we assume that σ is a function to the unit interval. Further, we assume that for each such function, there exists a special value $\epsilon_\sigma \in [0, 1]$, which represents indifference about similarity and dissimilarity in the sense of σ; ϵ_σ has the same role as the so called equilibrium in fuzzy set theory [18].

[5]This constraint may be relaxed. We leave this relaxation to future work.

Actually, the interpersonal link of A_i and A_j is not only affected by single pairs $\langle a_p, a_t \rangle$. Rather, we may simultaneously account for several tokens whose types are likewise related or unrelated to $\mathfrak{a}$. If, for example, $\mathfrak{a}$ is reproduced by A_i shortly after having been used by A_j, we expect that this is a hint to strengthen the link between A_i and A_j. However, if the distance between these usages increases, we expect an effect of equal rank only if the frequency of usages by A_j increases appropriately. Otherwise, we expect that the effect is an *unlearning* of the interpersonal link since the longer the distance, the less related these long-distance co-occurrences. Obviously, there is a trade-off between distance and similarity or relatedness: the longer the distance, the higher the similarity that is needed to get an alignment effect of equal rank. However, we also assume a certain boundary from which on the effect on alignment due to longer distance is negative. In a nutshell:

- The shorter the distances of usages of alike units by A_i and A_j, the higher the impact of these usages to learning an interpersonal link between both agents [19].
- Conversely, the longer the distances of usages of alike units by A_i and A_j, the higher the impact of these usages to unlearning an interpersonal link between both agents.

In order to account for these considerations in a generalized manner, we update $\hat{G}$ at t as follows:[6]

$$\hat{G}_t = \langle G_{0_t}, G_{1_{t-1}}, \ldots, G_{i_t}, \ldots, G_{k_{t-1}} \rangle \tag{8}$$

where

$$G_{0_t} = \begin{cases} (V_{0_{t-1}}, E_{0_{t-1}} \cup \{\{v, w\}\}, \mu_{0_{t-1}} \cup \{\langle \{v, w\}, \\ \qquad \sigma(\mathfrak{a}, \mathfrak{b}) \rangle\}, \mathcal{L}_0) : \\ \qquad \{v, w\} \notin E_{0_{t-1}} \\ (V_{0_{t-1}}, E_{0_{t-1}}, (\mu_{0_{t-1}} \setminus \{\langle \{v, w\}, \mu_{0_{t-1}}(\{v, w\}) \rangle\}) \\ \qquad \cup \{\langle \{v, w\}, \mu_{0_t}(\{v, w\}) \rangle\}, \mathcal{L}_0) : \\ \qquad \{v, w\} \in E_{0_{t-1}} \end{cases} \tag{9}$$

Henceforth, we call G_{0_t} the *interpersonal* subgraph of the multilog graph $\hat{G}_t$, while any subgraph G_{i_t}, $i \in \{1, \ldots, k\}$, is called *intrapersonal*. We now introduce the function μ_{0_t} as a parameter of our model. This is done to allow for varying learning functions that cope with learning as well as unlearning of interpersonal links. This can be done by means of the following scheme that conceives μ_{0_t} as a recursive learning function

$$\begin{aligned} \mu_{0_t}(\{v, w\}) = f[\mu_{0_{t-1}}(\{v, w\}); \{\langle \sigma(\mathfrak{a}, \mathfrak{b}), \delta(S_{a_p}, S_{a_t}) \rangle \mid \\ a_t \in A_i(\mathcal{M}) \wedge \tau(a_t) = \mathfrak{a} = \mathcal{L}(v) \wedge \\ a_p \in A_j(\mathcal{M}) \wedge \tau(a_p) = \mathfrak{b} = \mathcal{L}(w) \wedge \\ \theta(S_{a_p}) = \theta(S_{a_t}) \wedge 0 < p < t\}] \end{aligned} \tag{10}$$

which demands that μ_{0_t} is a function of the amount of association $\mu_{0_{t-1}}(\{v, w\})$ that has already been learned,

[6]The intrapersonal update of $G_{i_{t-1}}$ to G_{i_t} is defined in Sec. II-C.

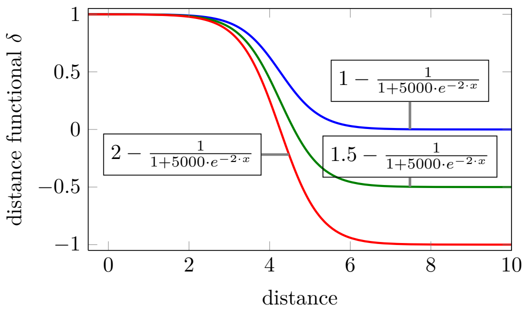

Figure 2. Three instances of Equation 12 for rewarding distances of alike and unalike elements in discourse.

of the similarities (or relatedness) $\sigma(\mathfrak{a},\mathfrak{b})$ of the types $\mathfrak{a}$ and $\mathfrak{b}$ that label v and w, respectively, and of the distances $\delta(S_{a_t}, S_{a_p})$ of the topic- or purpose-alike segments in which their tokens occur within the multilog under consideration. We assume that μ_{0_t} obeys the following axioms:

A1 $\forall v, w \in V_{0_t} : 0 \leq \mu_{0_t}(\{v,w\}) \in \mathbb{R}_0^+$.

A2 Of two tokens that are equally distant to a target token, the one of the same type as this target contributes more to the association of the corresponding vertices:
$\forall a_k \in A_i(\mathcal{M}) \forall a_l, a_m \in A_j(\mathcal{M}) : \mathcal{L}(u) = \tau(a_l) \wedge \mathcal{L}(v) = \tau(a_m) \wedge \mathcal{L}(w) = \tau(a_k) \wedge \delta(S_{a_k}, S_{a_l}) = \delta(S_{a_k}, S_{a_m}) \wedge \tau(a_l) \neq \tau(a_m) = \tau(a_k) \rightarrow \mu_{0_t}(\{u,w\}) \leq \mu_{0_t}(\{v,w\})$.

A3 Of two tokens that are of the same type as the target token, the one of shorter distance contributes more to the association of the corresponding vertices:
$\forall a_k \in A_i(\mathcal{M}) \forall a_l, a_m \in A_j(\mathcal{M}) : \mathcal{L}(u) = \tau(a_l) \wedge \mathcal{L}(v) = \tau(a_m) \wedge \mathcal{L}(w) = \tau(a_k) \wedge \delta(S_{a_k}, S_{a_l}) > \delta(S_{a_k}, S_{a_m}) \wedge \tau(a_l) = \tau(a_m) = \tau(a_k) \rightarrow \mu_{0_t}(\{u,w\}) \leq \mu_{0_t}(\{v,w\})$.

A first candidate instance of Scheme 10 that implements learning *and* unlearning is given by the following function (note that the choice of t determines a_t – see Figure 1):

$$\forall a_t \in A_i(\mathcal{M}), \tau(a_t) = \mathfrak{a} = \mathcal{L}(v) : \mu_{0_t}^{\alpha,\beta,\gamma}(\{v,w\}) = \max\Big(0, \mu_{0_{t-1}}(\{v,w\}) + \sum_{\substack{0<p<t, a_p \in A_j(\mathcal{M}),\\ \tau(a_p)=\mathfrak{b}, \mathcal{L}(w)=\mathfrak{b}, \theta(S_{a_p})=\theta(S_{a_t})}} \delta_{\alpha,\beta,\gamma}(S_{a_p}, S_{a_t})\kappa(\mathfrak{a},\mathfrak{b})\Big) \in \mathbb{R}_0^+ \quad (11)$$

where

$$\delta_{\alpha,\beta,\gamma}(S_{a_p}, S_{a_t}) = 1 - \frac{\gamma}{1+\alpha e^{-\beta(t-p)}} \quad (12)$$

and

$$\kappa(\mathfrak{a},\mathfrak{b}) = \begin{cases} \sigma(\mathfrak{a},\mathfrak{b}) : & \sigma(\mathfrak{a},\mathfrak{b}) \geq \epsilon_\sigma \\ -1(1-\sigma(\mathfrak{a},\mathfrak{b})) : & \sigma(\mathfrak{a},\mathfrak{b}) < \epsilon_\sigma \end{cases} \in [0,1] \quad (13)$$

Further, in the case of the present instance of Scheme 10,

Table II
Reference points of learning and unlearning according to Equation 11 under the condition that σ is a continuous function in $[0,1]$.

	$\sigma \rightarrow 1$	$\sigma \rightarrow 0$
$\delta_{\alpha,\beta,\gamma} \rightarrow 1$	$\kappa\delta_{\alpha,\beta,\gamma} \rightarrow 1$	$\kappa\delta_{\alpha,\beta,\gamma} \rightarrow -1$
$\delta_{\alpha,\beta,\gamma} \rightarrow -1$	$\kappa\delta_{\alpha,\beta,\gamma} \rightarrow -1$	$\kappa\delta_{\alpha,\beta,\gamma} \rightarrow 1$

we set:[7]

$$\sigma(\mathfrak{a},\mathfrak{b}) = \begin{cases} 1 : & \mathfrak{a} = \mathfrak{b} \\ 0 : & \mathfrak{a} \neq \mathfrak{b} \end{cases} \quad (14)$$

Note that Equation 13 assumes that $\sigma : L^2 \rightarrow [0,1]$ is a function to the unit interval. To exemplify this approach, suppose that $\alpha = 5,000$ and $\beta = \gamma = 2$. Now, if $x < 4.2586$, we get $\delta_{\alpha,\beta,\gamma}(S_{a_p}, S_{a_t}) > 0$, while for $x > 4.2586$, $\delta_{\alpha,\beta,\gamma}(S_{a_p}, S_{a_t}) < 0$ (see Figure 2). In the former case, μ is incremented by the focal pair $\langle a_p, a_t \rangle$ of tokens, while in the latter it is decremented. Of course, different tokens have varying effects on decrementing or incrementing μ which is finally balanced by the sum in Equation 11. In this sense, Equation 11 models learning as well as unlearning of associations. In any event, the less distant tokens $a_p \in A_j(\mathcal{M})$ that are the more similar to a_t, the higher the impact of these tokens in the sense of incrementing μ_{0_t}. On the other hand, the less alike these units and the higher their distance, the higher their impact on decrementing μ_{0_t}. Obviously, Equation 11 accounts for learning linguistic associations depending on the distances of the operative discourse segments as demanded by Scheme 10. Further, it may occur that several segments contribute to the same interpersonal link at time t. If A_i uses $\mathfrak{a}$ the first time, this contributes to a stronger link to A_j, the higher the number of preceding segments in short distance to S_{a_t} in which A_j used $\mathfrak{b} \sim \mathfrak{a}$. As mentioned before, this impact is weighted by the segments' distance to S_{a_t}: the greater this distance, the smaller the weight which finally becomes negative. As defined here, Equation 11 models priming based on the repetition of linguistic units. Of course, other functions that additionally account for structural similarities of $\mathfrak{a}$ and $\mathfrak{b}$, for their distance or for the distances of their constituents in semantic space (as modeled, e.g., by means of a terminological ontology [20]) may be used instead of Equation 14. In this sense, σ is a further central parameter of our model.

A central aspect of Equation 13 is that it treats short and long distances differently depending on the similarity or dissimilarity of the items in question: if the candidate type $\mathfrak{b}$ is similar to the target $\mathfrak{a}$ while their tokens are distant, the value of the product $\delta_{\alpha,\beta,\gamma}(S_{a_p}, S_{a_t})\kappa(\mathfrak{a},\mathfrak{b})$ is reduced. Conversely, if the distance is long (such that $\delta_{\alpha,\beta,\gamma}(S_{a_p}, S_{a_t}) < 0$) and the similarity below ϵ_σ, the effect is a reinforcement since $\delta_{\alpha,\beta,\gamma}(S_{a_p}, S_{a_t})$ with a negative

[7] Here, any other similarity function can be used, as long as it is a similarity function of the kind defined above.

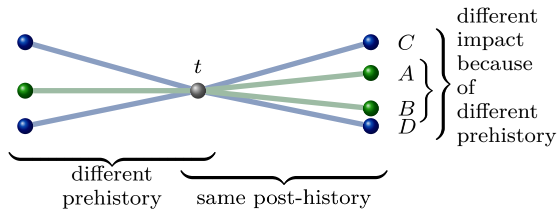

Figure 3. Impact of misalignment as a function of alignment history.

value (see Table II). In this sense, our model captures the distance as well as the similarity (or relatedness) of similar and dissimilar elements making the gradual build-up of a lexicon turn-by-turn a process in which interpersonal links originate, are reinforced or reduced in their strength, or even vanish. This generalization departs from [3] who implement learning by a monotone increasing function, which disregards unlearning.

One reason to reflect learning and unlearning in one model is to distinguish scenarios of non-alignment and anti-alignment [21]: in Figure 3, two idealized scenarios are shown for which we assume that they share their post-history but not their prehistory. Since the conversational partners A and B started from a more or less aligned dialog lexicon, we assume that the effect of their misalignment starting from point t is smaller than in the case of the interlocutors C and D who aligned only lately for a short period of time before misaligning again. In order to distinguish scenarios of this sort, we need to account for the conversational history of partners A and B who have a longer prehistory of alignment before starting to misalign. In other words, the longer the background of learned interpersonal associations among two agents the higher the "capital" that can be exploited before those agents get misaligned. Phenomena of this sort are modeled by defining μ_{0_t} as a recursive function that explores the prehistory of the focal point of time in the conversation of conversational partners.

In what follows, we consider three simplifications of this scheme: firstly, keeping repetition as the source of similarity, we consider the case that only such co-occurrences are counted that are located in a certain left-sided window of a_t (this is in accordance with predominant analyzes of lexical co-occurrences – see [22] for an overview). Secondly, we account for a sort of conversational learning of associations similar to co-occurrence analysis [23] by disregarding discontinuous segments of alike topics, but focusing on conversational segments as co-occurrence frames. Thirdly, we consider the case where we look only for the last repetition of $\mathfrak{a}$ by A_j irrespective of its distance to segment S_{a_t}.

Starting with the first alternative and using the definition of σ in Equation 14, we consider the following instance of Scheme 10:

$$\mu^{\omega}_{0_t}(\{v,w\}) = \begin{cases} \mu_{0_{t-1}}(\{v,w\}) + \sigma(\mathfrak{a},\mathfrak{b}) : \\ \quad L^{\mathcal{M},\omega}_t(v,w) = \{\mathfrak{a},\mathfrak{b}\} \\ \mu_{0_{t-1}}(\{v,w\}) : \\ L^{\mathcal{M},\omega}_t(v,w) = \emptyset \end{cases} \in \mathbb{R}^+ \quad (15)$$

where

$$\begin{aligned} L^{\mathcal{M},\omega}_t(v,w) = \{\mathfrak{a},\mathfrak{b} \mid\, & a_t, a_p \in \mathcal{A}(\mathcal{M}) \wedge \\ & \tau(a_t) = \mathfrak{a} = \mathcal{L}(v) = \mathcal{L}(w) = \mathfrak{b} = \tau(a_p) \wedge \\ & \theta(S_{a_t}) = \theta(S_{a_p}) \wedge \delta(S_{a_p}, S_{a_t}) = t - p \leq \omega\} \end{aligned} \quad (16)$$

This definition spans a *window* to the left of segment S_{a_t} whose size is determined by the parameter ω by incrementing the value of $\mu^{\omega}_{0_t}(\{v,w\})$ if there is a co-occurrence of the linguistic units under consideration within the frame spanned by ω. Note that the distances of the segments in these spans may vary so that Equation 16 implements a sort of "logical" co-occurrence span of variable length. Note that Equation 16 does not model unlearning.

It is obvious how one could extend this model along the lines of so called textual occurrences [22] in written texts that do no longer depend on the requirement that $\theta(S_{a_t}) = \theta(S_{a_p})$. In order to implement this, we need to omit the latter requirement in Equation 16. Depending on the nature of segments (say, for example, adjacency pairs or dialog games), this approach may be used to model lexical associations among conversational partners, similar to collocations in written communication. In any event, as long as we consider interpersonal associations, the counting frame must go beyond the boundaries of turns.

Finally, we consider a candidate of defining μ that includes unlearning, but focuses on a narrow context in contrast to Equation 11. In this alternative, only the last instance of the same type produced by A_j and repeated by A_i at time t is considered:

$$\mu^{\mathcal{M},\omega}_{0_t}(\{v,w\}) = \begin{cases} \frac{\omega}{t-p+1}(\mu_{0_{t-1}}(\{v,w\}) + \sigma(\mathfrak{a},\mathfrak{b})) : \\ |A^{\mathcal{M}}_t(v,w)| \neq \emptyset, p = \max_p A^{\mathcal{M}}_t(v,w) \\ 0 : |A^{\mathcal{M}}_t(v,w)| = \emptyset \end{cases} \in \mathbb{R}^+_0 \quad (17)$$

where

$$\begin{aligned} A^{\mathcal{M}}_t(v,w) = \{p < t \mid a_p, a_t \in \mathcal{A}(\mathcal{M}) \wedge \tau(a_t) = \mathfrak{a} = \mathcal{L}(v) \\ = \mathcal{L}(w) = \mathfrak{b} = \tau(a_p) \wedge \theta(S_{a_t}) = \theta(S_{a_p})\} \end{aligned} \quad (18)$$

This variant implements unlearning by means of the following distance function

$$\delta_\omega(S_{a_t}, S_{a_p}) = \frac{\omega}{t-p+1}\ ;\ \omega \in \mathbb{N} \quad (19)$$

Now, $\mu_{0_t}(\{v,w\})$ is decremented if $t - p + 1 > \omega$.

C. Intrapersonal Links

Self-priming is seen as a source of self-alignment that affects the capability of an interlocutor to align with his conversational partners.[8] Based on our model of interpersonal alignment, this process can be modeled straightforwardly as follows. When agent A_i, $i \in \{1, \ldots, k\}$, produces a_t at time t as a token of type $\mathcal{L}_i(v) = \mathfrak{a} = \tau(a_t)$ to express the purpose or topic $\theta(S_{a_t})$ of segment S_{a_t}, we relate this to previous productions of A_i in the context of segments of the same or alike topics. As a consequence, $G_{i_{t-1}}$ is adapted as follows to get G_{i_t} in

$$\hat{G}_t = \langle G_{0_t}, G_{1_{t-1}}, \ldots, G_{i_t}, \ldots, G_{k_{t-1}} \rangle \tag{20}$$

More specifically:

$$G_{i_t} = \begin{cases} (V_{i_{t-1}}, E_{i_{t-1}} \cup \{\{v,w\}\}, \nu_{i_{t-1}} \cup \{\langle\{v,w\}, \\ \quad \sigma(\mathfrak{a},\mathfrak{b})\rangle\}, \mathcal{L}_i) : \\ \quad \{v,w\} \notin E_{i_{t-1}} \\ (V_{i_{t-1}}, E_{i_{t-1}}, (\nu_{i_{t-1}} \setminus \{\langle\{v,w\}, \nu_{i_{t-1}}(\{v,w\})\rangle\}) \\ \quad \cup \{\langle\{v,w\}, \nu_{i_t}(\{v,w\})\rangle\}, \mathcal{L}_i) : \\ \quad \{v,w\} \in E_{i_{t-1}} \end{cases} \tag{21}$$

Basically, intrapersonal links can be generated by the same procedure as interpersonal links. However, instead of μ, we now denote the reward function by ν in order to allow the model for reflecting differences in intra- and interpersonal alignment. Thus, either ν is instantiated by μ – indicating that both mechanisms work the same way – or differently. In the latter case, the model is charged by another parameter.

III. Hyperedges as Models of Multi-agent Links

So far, we introduced intra- and interpersonal updates of linguistic items of counting units in multilogs. This model extends the dialog model of [3] by accounting for more than two conversational partners. However, this extension leaves open how to represent conversational structures based on subgroups of theses partners. In this section, we briefly sketch a graph model of such structures by example of "quadrolog", that is, multilogs of four partners as depicted in Figure 4. This figure enumerates a subset of interpersonal alignments starting from full non-alignment (a) to full alignment (h). Figure 4 distinguishes states of interpersonal alignment in terms of simple graphs whose vertices denote the sublexica of the corresponding multilog. Obviously, interpersonal alignment in multilogs gives rise to modeling clusters or subgroups of conversational partners beyond the level of dyads. In Figure 4(c), for example, a quadrolog is represented that is separated into two, non-overlapping subgroups of agents whose members are aligned with each other. In contrast to this, Figure 4(h) depicts a multilog, in which a single group of aligned partners emerged.

In order to give a graph-theoretical account of this kind of clustering, we utilize the notion of fuzzy hypergraphs [26]. This is done by inducing an agent alignment network $G_{\mathcal{A}_t}$ based on $\hat{G}_t$. More specifically, let $G_{0_t} = (V_{0_t}, E_{0_t})$ be the interpersonal subgraph of $\hat{G}_t$ at time t. Then, we define the weighted graph $G_{\mathcal{A}_t} = (\mathcal{A}, E_{\mathcal{A}_t}, \varphi_t)$ as the agent alignment network at time t such that $\mathcal{A}$ is the set of agents (see above) and

$$\forall \{A_i, A_j\} \in E_{\mathcal{A}_t} \exists v \in V_i \exists w \in V_j : \{v,w\} \in E_{0_t} \tag{22}$$

and

$$\varphi_t(\{A_i, A_j\}) = \frac{\sum_{\{v,w\} \in E_{0_t}, v \in V_i, w \in V_j} \frac{\mu_{0_t}(\{v,w\})}{t \cdot \sigma_{\max}}}{|V_i||V_j|} \in [0,1] \tag{23}$$

where $\sigma_{\max}$ is the maximum value that σ can assume. Starting from $G_{\mathcal{A}_t}$ we build a graph $G'_{\mathcal{A}_t} = (\mathcal{A}, E'_{\mathcal{A}_t}, \varphi_t)$ that filters out all edges, whose weight is smaller than ξ:

$$\forall \{A_i, A_j\} \in E'_{\mathcal{A}_t} \subseteq E_{\mathcal{A}_t} : \varphi_t(\{A_i, A_j\}) \geq \xi \tag{24}$$

Based on $G'_{\mathcal{A}_t}$, we induce the fuzzy hypergraph $\mathcal{G}_{\mathcal{A}_t} = (\mathcal{A}, \mathcal{E}_t)$ such that for all $\varepsilon_m \in \mathcal{E}_t = \{\varepsilon_1, \ldots, \varepsilon_o\}$, $\mathrm{supp}(\varepsilon_m)$ is a connected component of $G'_{\mathcal{A}_t}$ where the membership function ε_m is defined as follows: $\forall A_i \in \mathrm{supp}(\varepsilon_m)$:

$$\varepsilon_m(A_i) = \begin{cases} \frac{\sum_{A_j \in \mathrm{supp}(\varepsilon_m)} \varphi_t(\{A_i, A_j\})}{|\mathrm{supp}(\varepsilon_m)|-1} & : |\mathrm{supp}(\varepsilon_m)| > 1 \\ 0 & : |\mathrm{supp}(\varepsilon_m)| = 1 \end{cases} \tag{25}$$

Note that $\varepsilon_m(A_i) \in [0,1]$ and that $\mathcal{E}_t$ contains no duplicates. Further, for any hyperedge $\mathcal{E}_t \ni \varepsilon_i \subseteq \varepsilon_j \in \mathcal{E}_t \Rightarrow \varepsilon_i = \varepsilon_j$ and there are no $\varepsilon_i \neq \varepsilon_j$ such that $\varepsilon_i \cap \varepsilon_j \neq \emptyset$. That is, $\mathcal{E}_t$ is isomorph to the set of connected components of $G'_{\mathcal{A}_t}$ as induced by the threshold ξ of minimal interpersonal alignment.

Now, we are in a position to provide a first *measure of alignment in multilogs*:

$$a_t(\mathcal{M}) = \frac{\sum_{\varepsilon_m \in \mathcal{E}_t} \mathrm{hgt}(\varepsilon_m)}{|\mathcal{A}|} \in [0,1] \tag{26}$$

Note that in networks as shown in Figure 4a, $a_t(\mathcal{M}) = 0$. In contrast to this, $a_t(\mathcal{M}) = 1$, if and only if all agents are aligned to the maximum degree $\varphi_t(\{A_i, A_j\}) = 1$ so that they belong to the same fully connected component as shown in Figure 4h.

According to the measure in Equation 26, a multilog is the more aligned the higher the number of agents that are pairwise aligned by linguistic items of the higher order (as measured by φ_t). It is a central characteristic of our model to assume that this degree of alignment varies during a multilog by possibly reflecting a wide spectrum of alignment patterns as exemplified in Figure 4. To the best of our knowledge, such a variation has never been studied before. The apparatus introduced here aims at preparing such measurements that, so far, have been out of reach.

[8] The relation between self-alignment and other-alignment is not well studied, we know of two pertinent poster presentations: [24] report that self-alignment is even stronger than other-alignment. However, [25] found no difference between the two types of alignment.

(a) no alignm. (b) single dyadic alignment (c) pairwise dyadic alignment (d) overlapping pairwise dyadic (e) pairwise triadic alignment (f) discriminating alignment (g) transitive alignment (h) full alignm.

Figure 4. A subset of patterns of alignment in four-party discourse.

IV. Conclusion

Alignment has become a central topic in current research on dialog. Yet, not much is known about the spreading of alignment in multilogs, let alone approaches that try to quantify it. The present paper is a first step in this direction. Starting from a graph-theoretical model for lexical alignment in dialog (TiTAN), we first extended TiTAN with a dynamic component for unlearning. This allows us to capture and to distinguish various pathways of alignment in dyads. Based upon this extended alignment model, the last section provided a first account to measure the degree of alignment between more than two interlocutors. Multi-party discourse opens up a great range of possible configurations; some of them are depicted in Figure 4. The reconstruction of those patterns – most notably, schism and group size effects – are left to future work. The conceptual, formal model also has to be applied to natural multilogical language data (which are, by the way, few and far between).

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